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Surface motion of two canyons for incident SH waves by hybrid method

Wen-Shinn Shyu^{a*}, Tsung-Jen Teng^b, Chau-Shiung Yeh^c

^a*Department of Civil Engineering, National Pingtung University of Science and Technology, Pingtung, Taiwan*

^b*National Center for Research on Earthquake Engineering, Taipei, Taiwan*

^c*Department of Civil Engineering, National Taiwan University, Taipei, Taiwan*

Abstract

The responses to an incident plane SH wave on or near a surface included two canyons which is embedded in an elastic half-plane are investigated. A hybrid method combines the finite element method with series expansion is applied to solve the scattering problems in this study. A subregion encloses the two canyons with a semi-circular auxiliary boundary can be meshed by the finite element method. By using the transfinite interpolation (TFI) produces excellent grid mesh on the subregion. The hybrid method is successfully herein to solve the scattering problem.

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keywords : SH wave ; scattering ; canyon ; hybrid method ; transfinite interpolation

Nomenclature

C auxiliary boundary

* Corresponding author. Tel.: 886-8-7703202ext7194 ; fax: 886-8-7740122.

E-mail address: wsshuyu@mail.npust.edu.tw

C_s	shear wave speed
D^0	logical region
$[K]$	stiffness matrix
$[M]$	mass matrix
$\{P\}$	force vector
X	logical region coordinates
a	radius of canyon
d	distance between two canyon
x	physical region coordinates
$\{a\}$	displacement vector
$\{c\}$	determined coefficient vector
$\{u_y^0\}$	displacement vector
Ω	semi-infinite domain
Ω^0	irregular domain
η	dimensionless frequency
ω	circular frequency
θ	incident angle of SH wave
λ	shear wave length

1. Introduction

Local geological conditions can generate large amplifications and spatial variations in ground motion. Incident SH waves provide the simplest form of excitation. Pioneering work in this area was done in early '70s. Trifunac [1] obtained exact analytical solutions for the scattering of plane SH waves from a two-dimensional semi-cylindrical canyon. Then Wong and Trifunac [2] did so for a semi-elliptical canyon. Problems of the scattering of plane SH waves have been discussed over the last four decades. Focus on the canyon shapes, the models include multiple canyons [3], a truncated semi-cylindrical canyon [4], and a U-shaped canyon [5], among others. Then, hybrid method that combines domain method and series expansion provides a solution of scattering problem. Yeh *et al.* utilized the hybrid method to solve radiation problem in an elastic half-plane [6]. To verify the accuracy of the proposed method, a semi-elliptical canyon subjected to SH waves was investigated [7]. To overcome this disadvantage, transfinite interpolation [8] is adopted to mesh the domain of irregularity. A Case of an irregular shape of canyon subject to SH waves was also studied [9]. In this study, the hybrid method is utilized to solve the half-plane scattering problem with two equal circular canyons. This approach can deal with the scattering problems with different shapes of canyons.

2. Problem Description

Figure 1(a) presents a 2D model, which consists of two canyons embedded in the half-plane and impinged by a unit-amplitude plane SH wave with circular frequency ω , incident angle θ , and displacement in the y -direction. Let the elastic half-plane be divided into Ω and Ω^0 by a semi-circular auxiliary boundary C . Ω^0 is a finite domain that encloses the two canyons. For a special case, each semi-circular canyon with radius $a_1 = a_2 = a$, is considered in this study. The distance between corners of the two canyons is d . The sketch is shown in Fig 1(b).

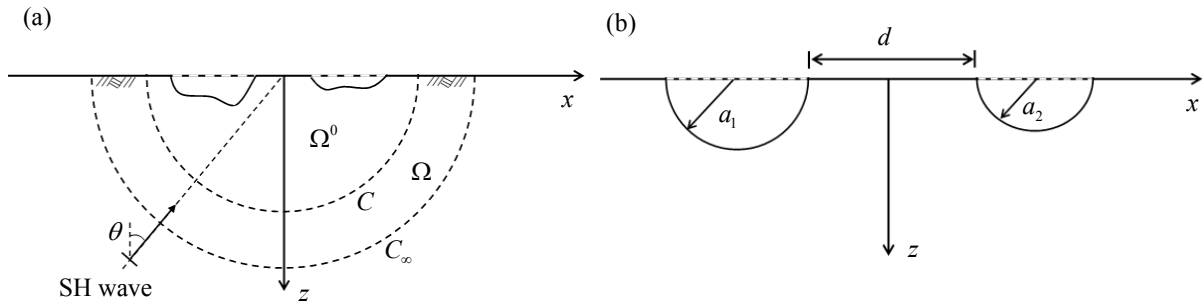


Fig. 1. (a) Model of SH wave scattering by two canyons and (b) two semi-circular canyons embedded on the half-space

3. Method of Solution

3.1. Hybrid method

The hybrid variational formulation is presented in the matrix equation

$$\begin{bmatrix} \mathbf{K}^{aa} - \omega^2 \mathbf{M}^{aa} & -\mathbf{K}^{ac} \\ -\mathbf{K}^{ca} & \mathbf{K}^{cc} \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \mathbf{c} \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}^a \\ -\mathbf{P}^c \end{Bmatrix} \quad (1)$$

where $[\mathbf{K}^{aa}]$ is global stiffness matrix, $[\mathbf{M}^{aa}]$ is global mass matrix and $\{\mathbf{P}^a\}$ is global free field traction vector of Ω^0 . $[\mathbf{K}^{ac}]$ is matrix formed along the boundary C , and $[\mathbf{K}^{ac}] = [\mathbf{K}^{ca}]$. $[\mathbf{K}^{cc}]$ is scattering matrix and $\{\mathbf{P}^c\}$ is the force vector formed by incident wave. $\{\mathbf{a}\}$ is the displacement field in each element can be represented by the vector of nodal displacement in Ω^0 . The scattered waves can be formulated as a series representation with determined coefficient vector $\{\mathbf{c}\}$. Therefore, the displacement in Ω^0 is

$$\{\mathbf{u}_y^0\} = \{\mathbf{a}\} \quad (2)$$

The detail description has been discussed by Yeh *et al.* [6].

3.2. Transfinite interpolation

The finite domain Ω^0 is mapped into the domain D^0 and the finite element meshes are discretized in the domain, as shown in Fig. 2. The domain Ω^0 is called the physical region, and the domain D^0 is called the logical region. The grid coordinates $\mathbf{X} = (X, Z)$ are chosen in the logical region, and \mathbf{x}_b^j , \mathbf{x}_t^j , \mathbf{x}_l^j and \mathbf{x}_r^j are the bottom, top, left, and right boundaries of the physical regions, respectively.

The transfinite interpolation (TFI) formula is introduced as a mapping function to calculate the coordinates of the physical region of the two canyons on the surface. The TFI formula is

$$\mathbf{x}(X, Z) = (1-Z)\mathbf{x}_b^j(X) + Z\mathbf{x}_t^j(X) + (1-X)\mathbf{x}_\ell^j(Z) + X\mathbf{x}_r^j(Z) - \{XZ\mathbf{x}_t^j(1) + X(1-Z)\mathbf{x}_b^j(1) + Z(1-X)\mathbf{x}_\ell^j(0) + (1-X)(1-Z)\mathbf{x}_r^j(0)\} \quad (3)$$

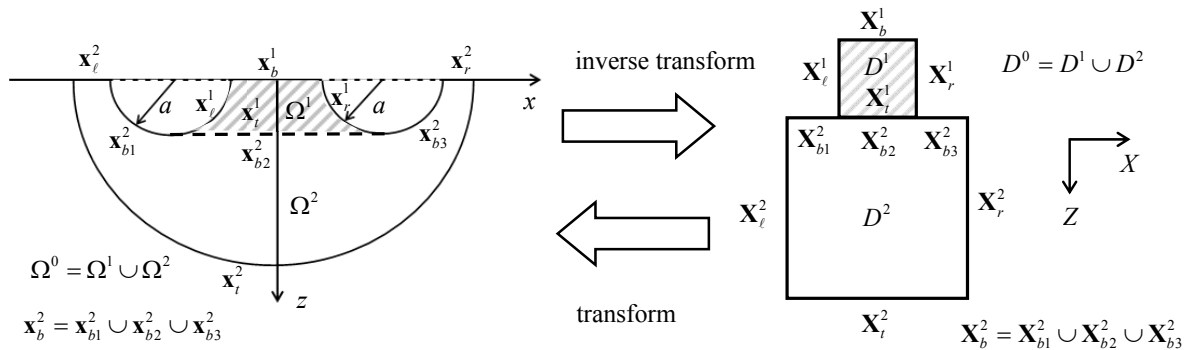


Fig. 2. Illustration of the region included two canyons and mapping functions

3.3. Results and discussions

In this study, a special case $a_1 = a_2 = a$ and $d/a = 1.0$, 2560 Q8 elements and 7905 nodes are used to mesh the domain Ω^0 . Figure 3(a) shows the mesh distributions on the surface irregularity included two canyons. The incident SH wave impinges with dimensionless frequency $\eta = 2a/\lambda = \omega a/\pi C_s$, where C_s is the shear wave speed of the half-plane. For $\eta = 1.0$, the surface displacement amplitudes of the canyons and neighborhood are compared with those obtained by null-field BIEM [3] in Fig 3(b). Figure 4 presents the surface displacement amplitudes for $d/a = 1.0$ vs. x/a and η with $\theta = 0$ and $\theta = \pi/2$. Figure 5 presents the surface displacement amplitudes for $d/a = 0.5$ vs. x/a and η with $\theta = 0$ and $\theta = \pi/2$. The surface displacement amplitudes in this study are compared with those obtained by single semi-circular canyon in Figs. 6(a) and 6(b). Solid line is the displacement amplitudes in this study and two dash lines are the displacement amplitudes of each single canyon moved to fit the positions of the two canyons. For the two cases shown in Figs. 6(a) and 6(b), one can see the displacement amplitudes of the two cases on illuminated side ($x/a < -d/2a - 1$) and shaded side ($x/a > d/2a + 1$) are similar. But the surface displacement amplitudes along the canyons vary rapidly, especially on the surface $-d/2a < x/a < d/2a$. It is clear to see the surface displacement amplitudes vs. different d/a shown in Figs. 7(a) and 7(b).

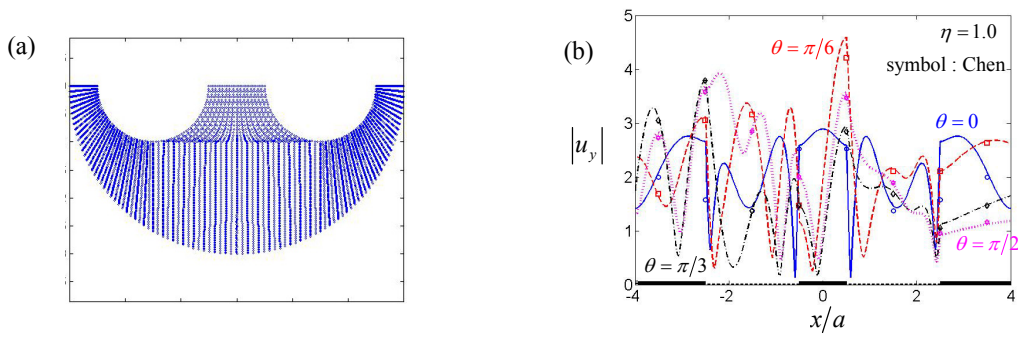


Fig. 3. (a) Mesh grid for the region included two canyons and (b) results in this study compared with Chen et. al. (2008).

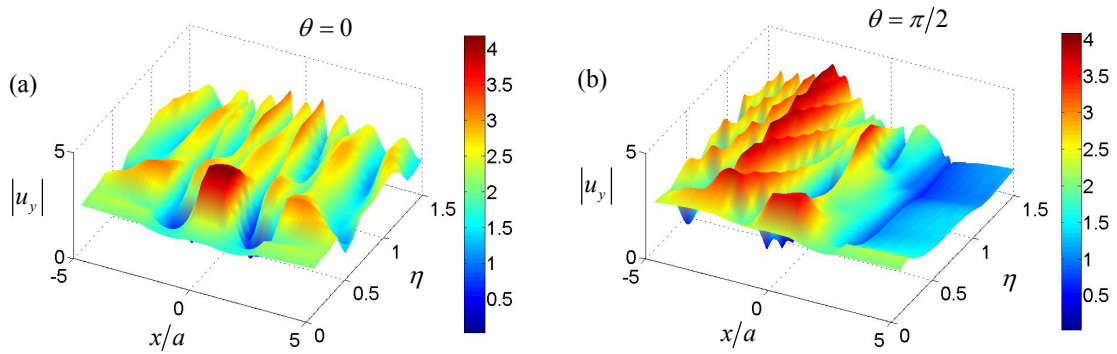


Fig. 4. Amplitude of surface displacement for $d/a = 1.0$ (a) $\theta = 0$ (b) $\theta = \pi/2$

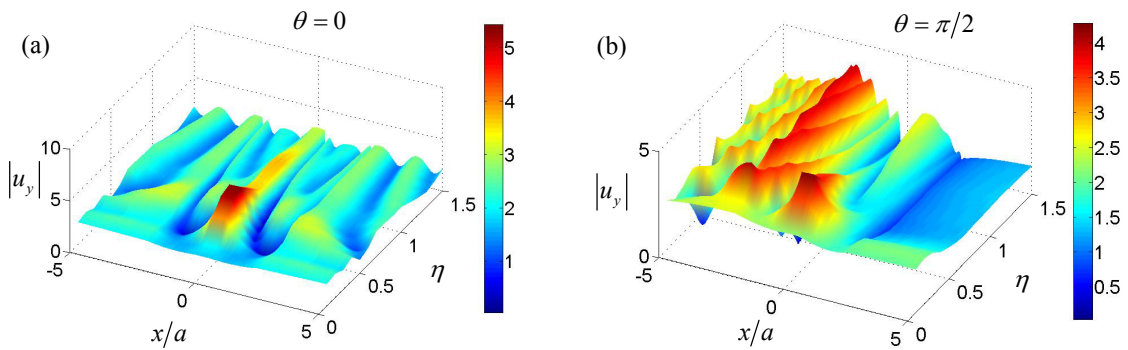


Fig. 5. Amplitude of surface displacement for $d/a = 0.5$ (a) $\theta = 0$ (b) $\theta = \pi/2$

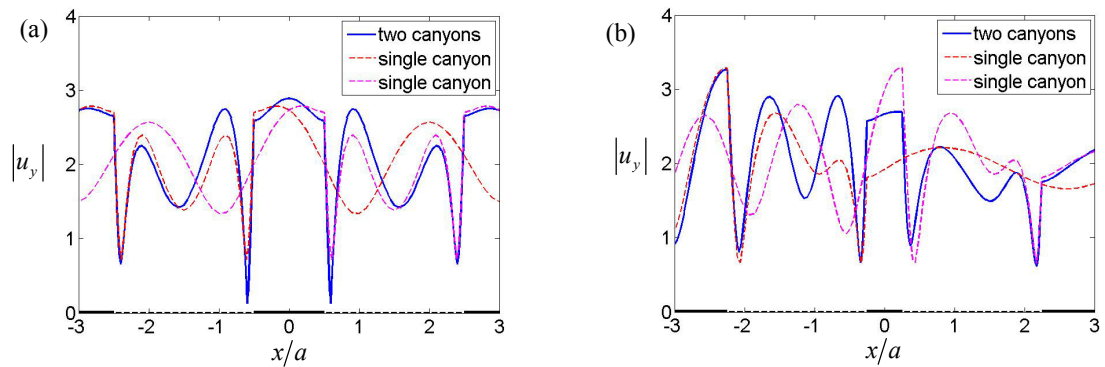


Fig. 6. Amplitude of surface displacement for (a) $d/a = 1.0, \eta = 1.0, \theta = 0$ and (b) $d/a = 0.5, \eta = 1.0, \theta = \pi/6$,

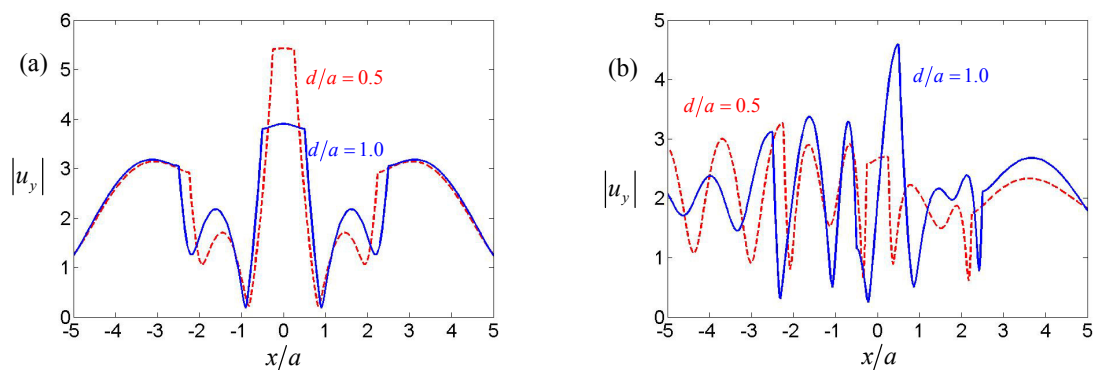


Fig. 7. Amplitude of surface displacement for (a) $\eta = 0.5, \theta = 0$ and (b) $\eta = 1.0, \theta = \pi/6$

4. Conclusion

This study presents results concerning the amplitude of surface displacement that is caused by incident SH waves on and near two canyons in an elastic half-plane. The displacement amplitudes of two equal semi-circular canyons are consistent with those obtained from BIEM. Results are presented for two canyons with different distances by using a hybrid method that combines TFI with series expansion. It shows the effect of multiple canyons is quite different from single canyon. The generated amplitudes could be helpful to explain the ground motion either observed or recorded during earthquakes near the similar topography. The hybrid method can be successfully applied to two canyons with more complex shapes.

Acknowledgements

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